13. Charlas en Uniandes 2005

Baldwin: in the sixties, four parts to logic - revolutions (forcing, $\diamond$, etc in Set Theory, Morley in Model Theory). East Coast (Yale): real structure theories, compactness, etc.

West Coast Model Theory: Cornell, Madison, Berkeley. Models of arbitrary cardinality, arbitrary theories. Quantifier elimination by fiat, MR, indiscernibles, compactness, combinatorial set theory, axiomatic set theory.

(Menciona un volumen de teoría de conjuntos del principio, tratando de entender lo que había hecho Cohen.)

Shelah's revolutions:

1. Classification of theories. Superstable? ω-stable? Thesis: 10 pages of results. Morley had conjectured that number of problems was not decreasing beyond $\aleph_1$... (!) Shelah's solution.
   - *a*) Trees - FO model theory from axiomatic set theory.
   - *b*) Systematizes analysis of structures.
2. Monster model or universal domain.
3. Dependence (forking). Families of combinatorial geometries.
4. Analyzing types locally.
   - *a*) Analyzing types locally
   - *b*) Relation between types: nonorthogonality, p-decomposition, **internality**.
5. Decompose a model into a tree of structures of card $|L|$.
6. Uncountable languages

Marxist Synthesis. Pillay (2000 - Urbana). 'There is only one Model Theory'.

Let Many Flowers Grow.

Models of Arithmetic, Finite Model Theory, Universal Algebra, Alternative Logics, Abstract Model Theory, Admissible Model Theory,.

13.1. Ideology

Can the study of structures of card $> 2^{\aleph_0}$ significantly impact the understanding of structures discovered in the 19th C?

Methodologically: YES.

In some sense (just as for large cardinals in set theory), Morley's theorem says NO. In some ways, Macintyre go along these lines.

Algebra of uncountable structures, as opposed to just combinatorics.

13.2. Math

AEC's - theories not logics.

Look at theories, instead of - Universal machine to understand mathematics

Structures are 'ensemblistes' - different from cats, continuous logic. Connections between these needs to be worked out.

In AEC, there is a notion of Monster Model $\mathcal{M}$, with a twist: look at subMODELS for homogeneity, not at arbitrary subsets of $\mathcal{M}$.

AP implies Monster Model.

Def of Galois type. [the diagram] Even in the absence of AP may work (!?!)

Under AP, Galois types are really orbits of automorphims of the Monster Model.

Syntactic types vs Galois types - Not necessarily the same, even under AP!

What are the key properties of syntactic types?

By *compactness*, a union of syntactic types is a syntactic type.

By *locality*, if $M = \bigcup M_i$, $p \in S(M)$, $p \neq q$, then there is some i st $p|M_i \neq q|M_i$.

By *tameness*, if $p, q \in S(M)$, $p \neq q$ then thre is $N \prec M$, $|N| < |M|$ with $p|N \neq q|N$.

Def of μ-tameness, Hanf number, Hanf number of omitting types.

Let $H(\kappa)$ be the Hanf number for AEC of language of LS κ. $H_1 = H(\aleph_0)$, $H_2 = H(H_1)$.

(All this makes sense by Shelah's presentation theorem.)

PC is intractable - by Silver's example.

$$H(\kappa) \leq \beth_{(2^\kappa)^+}$$

Two themes of categoricity in AECs

1. Eventual behavior.

2. What does categoricity in small ($\leq \aleph_\omega$ or even $\aleph_1$) cardinals imply?

Consequences of tameness:

(Shelah) If $\mathcal{K}$ is categorical in $\mu^+ \geq H_2$ then $\mathcal{K}$ is categorical in (H_2, μ^+).

(Grossberg, VanDieren) If $\mathcal{K}$ is stable in $\mu > H_1$, and tame (∞, α)-tame for $\alpha < H_1$, then $\mathcal{K}$ is stable in λ if $\lambda^\mu = \lambda$.

(Baldwin Kueker VanDieren) (∞, μ)-tame for $\mu < \kappa$ and κ-stable implies κ^+-stable.

If $\mathcal{K}$ is $(\infty, \aleph_0)$-local, then stable in all cardinals.

(∞, μ)-tame for $\mu \leq LS(K)$, and K is λ^+-categorical for $\lambda > LS(K)$ then cat in $[\lambda^+, \infty]$.

(Lessmann) $\aleph_0$-tame, $LS(K) = \aleph_0$, K is $\aleph_1$-cat, then cat in $[\aleph_1, \infty]$.

1. Zilber's quasiminimal excellent

2. (Shelah) result on tameness from 394.

The Hrushovski construction gives an AEC.

$(K, \leq, d)$. $\delta \to d = \min\{\delta(A') | \delta(A')A \subset A' \subset N ...\}$. d monotone, into the integers (!?!)

Get combinatorial geometry.

$a \in \mathrm{cl}_N(X)$ if $d(a/X) = 0$.

Grossberg-Kolesnikov look at abstract notion of dependence ... AND tameness

for every a, M there is a countable M_0 such that $d(a/M) = d(a/M_0)$.

The theorem: ext over indep pairs + free 2-amalg. + weak 3-amalg. + $\kappa(K)$ then the class is tame.

[The hypotheses are arbitrarily large models :( ]

13.3. The groups

Categorical short exact sequences ???

$$0 \to \mathbb{Z} \to V \to A \to 0$$

When $V = (\mathbb{C}, +)$ and $A = (\mathbb{C}, \cdot)$, then categorical all powers (Zilber)

Satisfies the hypotheses of BVZ. Plugging in Lessmann, have categoricity in all powers... if $\aleph_1$-categorical.

Zilber pairs cat (of certain seq) in all pairs implies arithmetic info.

Next task: pseudoexponentiation - how to extend this?

13.4. Non-tameness

Ab groups

$$0 \to \mathbb{Z} \to H \to G \to 0$$

Whitehead problem: if this splits, must G be free?
Shelah:

1. Whitehead Conjecture is independent
2. In ZFC thre is an $\aleph_1$-free non Whitehead group.

Not $(\aleph_1, \aleph_0)$-tame.

$$0 \to \mathbb{Z} \to H_1 \to G \leftarrow H_2 \leftarrow \mathbb{Z} \leftarrow 0$$

Can find structure that is μ_1-tame but not μ_2-tame if $\mu_2 > \mu_1$?
What if categorical?
Other: locality vs tameness? Find sufficient conditions for locality...
Tameness without amalgamation? Does that make any sense?
K admits closures if

$$\bigcap \{M \in K | X \subset M\} \in K$$

Thm: **(Baldwin)** If K admits closures, then there is K' with amalgamation and K' is no more tame than K.

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Good